# Market Structure and the Competitive Effects of Switching Costs*


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**Abstract**

We revisit the effects of switching costs on dynamic competition. We consider stationary Markovian strategies, with market shares being the state variable, and characterize a relatively simple Markov Perfect pricing equilibrium at which there is switching by some consumers at all times. For the case of low switching costs and infinitely lived consumers, we show that switching costs are pro-competitive in the long-run (steady state) while the overall effect in the short-run (transient state) depends on market structure. In particular, switching costs are anti-competitive in relatively concentrated markets, and pro-competitive otherwise.



**Keywords:** Switching costs, continuous-time model, Markov-perfect equilibrium, differential games, market concentration.

**JEL Codes:** C7, L1.


## 1 Introduction

Switching costs are widespread across a large range of products and services. For instance, customers must bear a cost when gathering information about a new product or service they would like to switch to, or when adopting a new technology that has limited compatibility with the old one. Other switching costs may arise from contractual arrangements (e.g. service contracts with a certain minimum term), or from loyalty programs (e.g. discount coupons or frequent flyer cards), to name just a few.

Switching costs face consumers with a potential lock-in effect which gives rise to dynamic market power. However, since past choices create inertia in consumers' future decisions,


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market share becomes a valuable asset that firms are willing to fight for. This gives rise to two countervailing incentives: on the one hand, firms want to charge high prices in order to exploit current customers, but on the other hand they also want to charge low prices in order to attract new ones. The conventional wisdom suggests that the former incentive dominates, so that switching costs give rise to anti-competitive effects.[1]

In this paper, we use a continuous-time infinite horizon game formulation to determine the extent to which the conventional wisdom relies on two features: (i) the finite horizon/finitely lived consumers assumptions, and (ii) the absence of switching in equilibrium. We consider stationary Markovian strategies, with market shares being the state variable, and characterize a relatively simple Markov Perfect pricing equilibrium at which there is switching by some consumers at all times.

For the case of low switching costs and infinitely lived consumers, we show that the conventional wisdom is only partially correct. Indeed, we find that switching costs are pro-competitive in the long-run (steady state) while the overall effect in the short-run (transient state) depends on market structure. These conclusions derive from the interplay of the countervailing incentives mentioned above, together with the presence of switching in equilibrium.

To understand the importance of switching in equilibrium, it is useful to interpret switching costs as a firm-subsidy when consumers are loyal to the firm (i.e., the firm can afford raising its price by the value of the switching cost without loosing consumers) or as a firm-tax when consumers are switching to the firm's product (i.e., the firm has to reduce its price by the amount of the switching cost in order to attract new consumers). For a large firm (which has more loyal consumers than consumers willing to switch into its product), the net effect of switching costs is that of a subsidy, whereas for a small firm the net effect of switching costs is that of a tax. Therefore, an increase in switching costs introduces a wedge in the pricing incentives of the two firms: while the large firm becomes less aggressive, the small firm becomes more so, thus inducing market shares to converge over time. In steady state firms become fully symmetric, so that the tax and the subsidy effects induced by an increase in switching costs cancel out for both firms.

However, in a dynamic setting, firms want to attract new consumers not simply as a source of current profits but also to exploit them in the future. Since the value of new customers is greater the higher the switching costs, an increase in switching costs fosters more competitive outcomes in steady state.[2]

In contrast, the short-run effect of an increase in switching costs is ambiguous. While

[1] This conclusion is also supported on a series of models with either a finite horizon (e.g. Klemperer (1987a)) or with finitely lived consumers (e.g. Padilla (1995)), in all of which switching does not occur in equilibrium. Other papers include Klemperer (1987b), Farrell and Shapiro (1988), To (1995) and Villas-Boas (2006), among others. Klemperer (1995) and Farrell and Klemperer (2007) provide a survey of this literature.

[2] There are other recent papers showing the potential pro-competitive effect of switching costs. See Viard (2007), Cabral (2011,2012), Dubé *et al.* (2009), Shi *et al.* (2006), Doganoglu (2010), Arie and Grieco (2013), and Rhodes (2013). Our paper differs from this literature, which features overlapping generation models for finitely lived consumers and discrete time models of dynamic price competition.

higher switching costs reduce the prices charged by the small firm, the effect on the large firm's pricing incentives depends upon the level of market dominance. With strong market dominance, an increase in switching costs induces higher prices by the dominant firm. This corresponds with the conventional wisdom. However, under weak market dominance, an increase in switching costs induces lower prices by both firms. In this situation, switching costs are pro-competitive also in the short-run.

# 2 The Model

We consider a market in which two firms compete to provide a service which is demanded continuously over time. Firms have identical marginal costs normalized to zero. There is a unit mass of infinitely lived consumers. We assume that all consumers are served. Letting $x_i(t)$ denote the market share of firm $i \in \{1, 2\}$, this implies $x_1(t) + x_2(t) = 1$.

Switching opportunities for consumers take place over time according to independent Poisson processes with unit rate,[3] i.e., in the interval $(t, t + dt)$, the expected fraction of consumers considering switching or not between firms is $dt$. We assume that consumers cannot anticipate infinite equilibrium price trajectories and thus can only react to current prices.[4] Conditional on having the opportunity to switch, $q_{ji} \in (0, 1)$ denotes the probability with which a customer currently served by firm $j$ switches to firm $i$. Accordingly, $q_{jj} = 1 - q_{ji}$ is the probability that a customer already served by firm $j$ maintains this relationship. Firm $i$'s net (expected) change in market share in the infinitesimal time interval $(t, t + dt]$ can be expressed as

$$x_i(t + dt) - x_i(t) = q_{ji}x_j(t)dt - (1 - q_{ii})x_i(t)dt, \tag{1}$$

i.e., the net (expected) change in market share is equal to the expected number of customers that firm $i$ steals from firm $j$, minus the customers that firm $j$ steals from firm $i$. The revenue accrued in the infinitesimal time interval $(t, t + dt]$ is the sum of $p_ix_i(t)dt$ (i.e., revenue from current customers) and $p_i[q_{ji}x_j(t) - (1 - q_{ii})x_i(t)]dt$ (i.e., revenue gain/loss from new/old customers). Hence, the rate at which revenue is accrued by firm $i$, say $\pi_i(t)$, can be written as

$$\pi_i(t) = p_ix_i(t) + p_i[q_{ji}x_j(t) - (1 - q_{ii})x_i(t)]. \tag{2}$$

In order to characterize the switching probabilities, we assume a discrete choice model in which the net surplus from product $i \in \{1, 2\}$ at time $t > 0$ is of the form

$$u_i(t) = v_i(t) - p_i(t)$$

[3] The analysis is robust to arrival rates different from one. However, this would add an additional parameter in the model, with only a scaling effect. One could instead envisage a model in which the arrival rate is a function of firms' prices. However, this would further complicate the analysis, and it is out of the scope of the current paper.

[4] The main conclusions of the paper are preserved if we allowed for more sophisticated consumers. See Section 3 for further comments on this issue. See Fabra and García (2013).

wherein we make the following standing assumption:[5]

*Assumption:* The collection $\{v_i(t) : t > 0\}$ is *i.i.d.* and $v_i(t) - v_j(t)$ is uniformly distributed in $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

When an opportunity to switch arises for a given customer, he would opt for firm $i$ (if currently served by firm $j$) provided that

$$u_i - \frac{s}{2} = v_i - p_i - \frac{s}{2} > u_j = v_j - p_j$$

where $\frac{s}{2}$ is the switching cost incurred ($s < 1$). Hence, the probability that such a customer served by firm $j$ switches to firm $i$, $q_{ji}$, is given by

$$q_{ji} = \Pr\left(v_j - v_i < -\frac{s}{2} + p_j - p_i\right) = \frac{1}{2}(1-s) - p_i + p_j$$

where we assume $p_i - p_j \in \left[-\frac{1}{2}(1-s), \frac{1}{2}(1+s)\right]$. Conversely, if firm $i$ serves the selected consumer, he will maintain this relationship if

$$u_i = v_i - p_i > u_j = v_j - p_j - \frac{s}{2}.$$

Hence, the probability $q_{ii}$ that a customer already served by firm $i$ maintains this relationship is[6]

$$q_{ii} = \Pr\left(v_j - v_i < \frac{s}{2} - p_i + p_j\right) = \frac{1}{2}(1+s) - p_i + p_j.$$

Substituting $q_{ji}$ and $q_{ii}$ into (1) and taking the limit, as $dt \to 0$, we obtain

$$\dot{x}_i(t) = -x_i(t)(1-s) + \frac{1-s}{2} - p_i + p_j.$$

Substituting $q_{ji}$ and $q_{ii}$ into (2), we obtain the rate at which revenue is accrued by firms 1 and 2,

$$\begin{aligned} \pi_1(t) &= p_1\left(x_1(t)s + \frac{1-s}{2} - p_1 + p_2\right) \\ \pi_2(t) &= p_2\left(-x_1(t)s + \frac{1+s}{2} + p_1 - p_2\right). \end{aligned}$$

Given the assumption of full market coverage, payoff relevant histories are subsumed in the state variable $x_1 \in [0,1]$. Assume a discount rate $\rho > 0$. A stationary Markovian pricing policy is a map $p_i : [0,1] \to [0, \bar{p}]$ where $\bar{p} > 0$ is the maximum price ensuring full-market coverage. We restrict our attention to the set of continuous and bounded Markovian pricing

[5] Note this assumption is consistent with Hotelling's model of product differentiation with product varieties at the extremes of a linear city uniformly distributed in $[0, \frac{1}{2}]$. Results are robust to allowing for more general distributions.

[6] Note that $q_{ii} \geq q_{ji}$ reflects the fact that, for given prices, firm $i$ is more likely to retain a randomly chosen current customer than to "steal" one from firm $j$.

policies, say $\mathcal{P}$. For a given strategy combination $(p_i, p_j) \in \mathcal{P} \times \mathcal{P}$ and initial condition, $x_1(\tau) \in [0,1]$ and $\tau < \infty$, the value function is defined as

$$V_i^{(p_i,p_j)}(x_1(\tau)) = \int_\tau^\infty e^{-\rho t}\pi_i(p_i(x_1(t)), p_j(x_1(t)), x_1(t))dt\cdot$$

A stationary Markovian strategy combination $(p_i^*, p_j^*) \in \mathcal{P} \times \mathcal{P}$ is a Markov Perfect equilibrium (MPE) if and only if

$$V_i^{(p_i^*,p_j^*)}(x_1(\tau)) \geq V_i^{(p_i,p_j^*)}(x_1(\tau)),$$

for all $p_i \in \mathcal{P}, i \in \{1,2\}$, $x_1(\tau) \in [0,1]$ and $\tau < \infty$.

## 2.1 Equilibrium Analysis

The following Proposition characterizes equilibrium pricing in this game.

**Proposition 1** *Assuming $s < \frac{3}{5}$,the unique Markov Perfect Equilibrium in affine pricing strategies is:*[7]

$$\begin{aligned} p_1(x_1) &= \tfrac{1}{3}(s-a)\left(x_1 - \tfrac{1}{2}\right) + p^* \\ p_2(x_1) &= -\tfrac{1}{3}(s-a)\left(x_1 - \tfrac{1}{2}\right) + p^* \end{aligned}$$

*where $a \in \left(0, \frac{s}{2}\right)$ is the smallest root of the quadratic equation*

$$2a^2 - 3\left(2 + \rho - \frac{7}{9}s\right)a + \frac{2}{3}s^2 = 0, \tag{3}$$

*and*

$$p^* = \frac{1}{2} + \frac{a}{2(1+\rho)} - \frac{s}{3}\frac{1+\frac{a}{2}}{1+\rho}\cdot$$

In the proof of Proposition 1 we make use of the notion of a Hamiltonian (see Dockner *et al.* (2000)), that is:

$$\mathcal{H}_i = e^{-\rho t}[\pi_i + \lambda_i \dot{x}_1],$$

for $i \in \{1,2\}$ , where $\lambda_i = \frac{\partial V_i}{\partial x_1}$ is the co-state variable. The necessary and sufficient conditions for a Markov Perfect equilibrium are

$$\frac{\partial \pi_i}{\partial p_i} = -\lambda_i \frac{\partial \dot{x}_1}{\partial p_i}, \tag{4}$$

which capture the inter-temporal trade-offs inherent in equilibrium pricing, i.e., marginal revenue equals the (marginal) opportunity cost (value loss) associated with market share reduction, and the Hamilton-Jacobi equations

$$-\frac{\partial \mathcal{H}_i}{\partial x_1} - \frac{\partial \mathcal{H}_i}{\partial p_j}\frac{\partial p_j}{\partial x_1} = \dot{\lambda}_i - \rho\lambda_i. \tag{5}$$

[7]Other MPE in non-linear strategies may exist. However, a complete characterization of MPE is beyond the scope of this paper.

In the proof of Proposition 1 we show that the system of partial differential equations (4) and (5) has a closed-form solution.

Note that condition (4) gives rise to a sort of "instantaneous" best reply functions

$$\begin{array}{rcl} R_1\left(p_2\right) & = & \frac{1}{2}\left(p_2+s\left(x_1-\frac{1}{2}\right)+\frac{1}{2}\right)-\frac{\lambda_1}{2}, \\ R_2\left(p_1\right) & = & \frac{1}{2}\left(p_1-s\left(x_1-\frac{1}{2}\right)+\frac{1}{2}\right)+\frac{\lambda_2}{2}\cdot \end{array}$$

Therefore, as compared to a static setting (in which $\lambda_i$ would equal zero), firms' best reply functions in the dynamic setting shift in, thus implying that equilibrium prices are lower. This is a direct consequence of the fact that firms compete more aggressively in order to attract new customers, as these will become loyal in the future. In the proof of Proposition 1 we show that $\lambda_1 = ax_1 + b > -\lambda_2 = -ax_1 + b > 0$. Hence, in the dynamic setting, the large firm behaves less aggressively than the small firm, i.e., $R_1(p) > R_2(p)$, thus implying that the large firm's equilibrium price is higher than that of the small firm, regardless of the value of $s$,

$$p_1(x_1) - p_2(x_2) = \frac{2}{3}(s-a)\left(x_1 - \frac{1}{2}\right) > 0\cdot$$

Concerning dynamics, the fact that the large firm has the high price implies that the large firm concedes market share in favour of the smaller one. Therefore, market share asymmetries fade away over time. In particular, the equilibrium state dynamics are described by

$$\begin{array}{rcl} \dot{x}_1(t) & = & -x_1(t)(1-s) + \dfrac{1-s}{2} - p_1(x_1(t)) + p_2(x_1(t)) \\ & = & -\left(x_1(t) - \dfrac{1}{2}\right)\left(1 - \dfrac{s+2a}{3}\right) < 0, \end{array}$$

whose solution is

$$x_1(t) = x_1(0)e^{-\left(1-\frac{s+2a}{3}\right)t} + \frac{1}{2}\cdot$$

Furthermore, as the large firm loses market share, its incentives to price high diminish, and competition becomes more intense. Hence, the average price in the market is *decreasing* over time. Let $p(t) = p_1(x_1(t))x_1(t) + p_2(x_1(t))x_2(t)$ denote the average price charged in the market. After some algebra, it follows that

$$\dot{p}(t) = \left[\frac{4}{3}(s-a)\left(x_1 - \frac{1}{2}\right)\right]\dot{x}_1 < 0\cdot$$

Note that in steady state market shares become symmetric, as $\lim_{t\to\infty} x_1(t) = \frac{1}{2}$, and both firms' equilibrium prices converge to $p^*$.

## 2.2 Comparative Dynamics

We end the analysis by performing comparative dynamics of equilibrium outcomes as switching costs increase.

**Lemma 1** *In the short-run:*

*(i) An increase in* $s$ *reduces the price charged by the small firm;*

*(ii) There exists* $\widehat{x}_1(s) > 1/2$ *such that an increase in* $s$ *reduces the price charged by the large firm if and only if* $x_1 < \widehat{x}_1(s)$.

*(iii) In the short-run, there exists* $\widetilde{x}_1 > \widehat{x}_1(s)$ *such that an increase in* $s$ *reduces the average market price if and only if* $x_1 < \widetilde{x}_1$.

When switching costs increase, the small firm prices more aggressively as the customers served by the large firm would not switch otherwise. Also, the value of attracting customers is greater the higher $s$ as those customers will be locked-in in the future.

In contrast, the effects of an increase in $s$ on the large firm's pricing incentives are ambiguous. On the one hand, the large firm can afford charging a higher price to its customers given that they are protected by a higher switching cost. However, an increase in $s$ also enhances the value of attracting new customers. Since the incentives to charge higher prices today are greater the larger the firm's market share, there exists a critical market share $\widehat{x}_1(s)$ below (above) which the long-run (short-run) effect dominates, so that the price charged by the large firm decreases (increases) in $s$.

Although $x_1 < \widehat{x}_1(s)$ is sufficient for the average market price to go down as $s$ goes up, it is by no means necessary. In particular, the average market price starts decreasing in $s$ even before the large firm's market share falls down to $\widehat{x}_1(s)$ since the lower prices charged by the small firm more than compensate for the higher prices charged by the large one.

Switching costs induce other anti-competitive effects. In particular, they slow down the transition to a symmetric market structure, and hence lead to a lower rate of decline in average prices. This anti-competitive effect arises regardless of the degree of market dominance.

**Lemma 2** *An increase in switching costs* $s$*:*

*(i) reduces the rate of decline of average prices and*

*(ii) delays the transition to the steady state.*

In the long-run, switching costs are pro-competitive: the *higher* the switching cost, the *lower* the equilibrium price in steady state. Indeed, once firms' market shares have become fully symmetric, only the incentive to attract new customers plays a role. Hence, an increase in $s$, which increases the future value of current sales, makes competition fiercer and thus lowers equilibrium prices.

**Proposition 2** *In steady state, an increase in switching costs reduces prices.*

# 3 Conclusions and Discussion

We have shown that (relatively low) switching costs can be pro-competitive in a model with infinitely lived consumers. In a Markov Perfect equilibrium, the dominant firm concedes market share by charging higher prices to current customers. As the market becomes less concentrated, price competition becomes fiercer. The average price charged in the market is decreasing over time and in the long-run equilibrium prices are decreasing in the level of switching costs.

However, in the short-run, switching costs can have an ambiguous effect on prices, depending on market structure. When market shares are sufficiently asymmetric, the increase in the price charged by the large firm outweighs the reduction in the price charged by the smaller competitor. It is only when firms' market shares have become sufficiently symmetric over time that switching costs become unambiguously pro-competitive.

For tractability, we have focused on linear equilibria, but believe that other equilibria - should they exist - would depict similar properties. If switching costs increase, the static effects cancel out when firms are symmetric, and this would remain true regardless of the class of MPE considered. Switching costs are pro-competitive since only the dynamic incentive to attract new consumers prevails. Again, since this incentive does not depend on firms' current market shares, it would arise regardless of how these enter into firms' strategies. Other MPE could have different rates of convergence to the steady state, but they would share the qualitative features of the linear equilibrium characterized in this paper.[8]

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# Appendix: Proofs of Lemmas and Propositions

## Proof of Proposition 1

The Hamiltonians are

$$\mathcal{H}_i = e^{-\rho t}[\pi_i + \lambda_i \dot{x}_1],$$

for $i = 1, 2$. The Hamiltonians are strictly concave so that first order conditions for MPE are also sufficient (see Dockner et al. (2000)),

$$\frac{\partial \mathcal{H}_i}{\partial p_i} = 0,$$

$$-\frac{\partial \mathcal{H}_i}{\partial x_1} - \frac{\partial \mathcal{H}_i}{\partial p_j}\frac{\partial p_j}{\partial x_1} = \dot{\lambda}_i - \rho\lambda_i,$$

for $i = 1, 2$. These respectively lead to:

$$p_1 = \frac{1}{2}\left(p_2 + s\left(x_1 - \frac{1}{2}\right) + \frac{1}{2} - \lambda_1\right) \tag{A.1}$$

$$-sp_1 + (1-s)\lambda_1 - (p_1 + \lambda_1)\frac{\partial p_2}{\partial x_1} = \dot{\lambda}_1 - \rho\lambda_1 \tag{A.2}$$

$$p_2 = \frac{1}{2}\left(p_1 - s\left(x_1 - \frac{1}{2}\right) + \frac{1}{2} + \lambda_2\right) \tag{A.3}$$

$$sp_2 + (1-s)\lambda_2 - (p_2 - \lambda_2)\frac{\partial p_1}{\partial x_1} = \dot{\lambda}_2 - \rho\lambda_2. \tag{A.4}$$

Equations (A.1) and (A.3) are firms' best reply functions. Using them we can obtain equilibrium prices,

$$\begin{aligned} p_1 &= \frac{s}{3}\left(x_1 - \frac{1}{2}\right) + \frac{1}{2} + \frac{1}{3}(\lambda_2 - 2\lambda_1) \\ p_2 &= -\frac{s}{3}\left(x_1 - \frac{1}{2}\right) + \frac{1}{2} + \frac{1}{3}(2\lambda_2 - \lambda_1). \end{aligned}$$

Thus,

$$\frac{\partial p_2}{\partial x_1} = -\frac{\partial p_1}{\partial x_1} = -\frac{s}{3}.$$

Substituting into (A.2) and (A.4) we obtain

$$\begin{aligned} \frac{2s}{3}p_2 + (1 - \frac{2s}{3})\lambda_2 &= \dot{\lambda}_2 - \rho\lambda_2 \\ -\frac{2s}{3}p_1 + (1 - \frac{2s}{3})\lambda_1 &= \dot{\lambda}_1 - \rho\lambda_1. \end{aligned}$$

We solve this system of differential equations by the method of undetermined coefficients. Assume $\lambda_i = a_i(x_1 - \frac{1}{2}) + b_i$ for $i = 1, 2$. Substitution into the last equation yields

$$\begin{gathered} -\tfrac{2s}{3}\left(\tfrac{s}{3}(x_1 - \tfrac{1}{2}) + \tfrac{1}{2} + \tfrac{1}{3}(a_2(x_1 - \tfrac{1}{2}) + b_2 - 2a_1(x_1 - \tfrac{1}{2}) - 2b_1)\right) + (1 - \tfrac{2s}{3})(a_1(x_1 - \tfrac{1}{2}) + b_1) \\ = \\ a_1\dot{x}_1 - \rho(a_1(x_1 - \tfrac{1}{2}) + b_1) \\ = \\ a_1\left(-x_1(1-s) + \tfrac{1-s}{2} - p_1 + p_2\right) - \rho a_1(x_1 - \tfrac{1}{2}) - \rho b_1 \\ = \\ a_1\left(-x_1(1-s) + \tfrac{1-s}{2} - \tfrac{2s}{3}(x_1 - \tfrac{1}{2}) + \tfrac{\lambda_1 + \lambda_2}{3}\right) - \rho a_1(x_1 - \tfrac{1}{2}) - \rho b_1 \\ = \\ a_1\left(-(1-s)(x_1 - \tfrac{1}{2}) - \tfrac{2s}{3}(x_1 - \tfrac{1}{2}) + \tfrac{a_1(x_1 - \frac{1}{2}) + b_1 + a_2(x_1 - \frac{1}{2}) + b_2}{3}\right) - \rho a_1(x_1 - \tfrac{1}{2}) - \rho b_1 \end{gathered}$$

This results in the following two equations:

$$-\frac{2}{9}s^2 + \frac{2}{9}s(2a_1 - a_2) + (1 - \frac{2s}{3})a_1 = -(1 - \frac{s}{3})a_1 + \frac{1}{3}(a_1 + a_2)a_1 - \rho a_1 \tag{A.5}$$

$$-\frac{s}{3}-\frac{2s}{9}(b_2-2b_1)+b_1(1-\frac{2s}{3})=\frac{1}{3}(b_1+b_2)a_1+\frac{a_1}{2}(1-\frac{s}{3})-\rho b_1 \tag{A.6}$$

In a similar fashion,

$$\tfrac{2s}{3}\left(-\tfrac{s}{3}\left(x_1-\tfrac{1}{2}\right)+\tfrac{1}{2}+\tfrac{1}{3}(2a_2\left(x_1-\tfrac{1}{2}\right)+2b_2-a_1\left(x_1-\tfrac{1}{2}\right)-b_1)\right)+(1-\tfrac{2s}{3})(a_2\left(x_1-\tfrac{1}{2}\right)+b_2)$$
$$=$$
$$a_2\dot{x}_1-\rho(a_2\left(x_1-\tfrac{1}{2}\right)+b_2)$$
$$=$$
$$a_2\left(-x_1(1-s)+\tfrac{1-s}{2}-\tfrac{2s}{3}(x_1-\tfrac{1}{2})+\tfrac{a_1\left(x_1-\frac{1}{2}\right)+b_1+a_2\left(x_1-\frac{1}{2}\right)+b_2}{3}\right)-\rho a_2\left(x_1-\tfrac{1}{2}\right)-\rho b_2$$
$$=$$
$$a_2\left(-(1-s)(x_1-\tfrac{1}{2})-\tfrac{2s}{3}(x_1-\tfrac{1}{2})+\tfrac{a_1\left(x_1-\frac{1}{2}\right)+b_1+a_2\left(x_1-\frac{1}{2}\right)+b_2}{3}\right)-\rho a_2\left(x_1-\tfrac{1}{2}\right)-\rho b_2$$

We obtain two additional equations:

$$-\frac{2}{9}s^2+\frac{2}{9}s(2a_2-a_1)+(1-\frac{2s}{3})a_2=-(1-\frac{s}{3})a_2+\frac{1}{3}(a_1+a_2)a_2-\rho a_2 \tag{A.7}$$

$$\frac{s}{3}+\frac{2s}{9}(2b_2-b_1)+b_2(1-\frac{2s}{3})=\frac{1}{3}(b_1+b_2)a_2+\frac{a_2}{2}(1-\frac{s}{3})-\rho b_2 \tag{A.8}$$

Thus, substracting (A.5) from (A.7) we get:

$$\left[1-\frac{s}{3}-\frac{1}{3}(a_1+a_2)+\frac{2}{9}s+1-\frac{2s}{3}+\rho\right](a_1-a_2)=0\cdot$$

Hence, $a_1-a_2=0$. Letting $a_1=a_2=a$, we solve the quadratic equation implicit in (A.5):

$$2a^2-3\left(2+\rho-\frac{7}{9}s\right)a+\frac{2}{3}s^2=0\cdot$$

Then (A.6) and (A.8) imply $b_1+b_2=0$ and

$$b_1=\frac{s(1+\frac{a}{2})}{3(1+\rho)}-\frac{a}{2(1+\rho)}.$$

So the equilibrium strategies can be rewritten as

$$\begin{aligned} p_1 &= \frac{s}{3}\left(x_1-\frac{1}{2}\right)+\frac{1}{2}+\frac{1}{3}(\lambda_2-2\lambda_1),\\ p_2 &= -\frac{s}{3}\left(x_1-\frac{1}{2}\right)+\frac{1}{2}+\frac{1}{3}(2\lambda_2-\lambda_1)\cdot \end{aligned}$$

Or equivalently,

$$\begin{aligned} p_1 &= \frac{s-a}{3}\left(x_1-\frac{1}{2}\right)+\frac{1}{2}+\frac{a}{2(1+\rho)}-\frac{s(1+\frac{a}{2})}{3(1+\rho)}\\ p_2 &= -\frac{s-a}{3}\left(x_1-\frac{1}{2}\right)+\frac{1}{2}+\frac{a}{2(1+\rho)}-\frac{s(1+\frac{a}{2})}{3(1+\rho)}. \end{aligned}$$

Finally, we note that given the assumption $s<\frac{3}{5}$ the pricing policies satisfy

$$p_1(x_1)-p_2(x_1)=\frac{2}{3}(s-a)\left(x_1-\frac{1}{2}\right)\in\left(-\frac{1-s}{2},\frac{1-s}{2}\right),$$

so that $q_0,q_1\in(0,1)$.

## Proof of Lemma 1

We first note that implicit differentiation in (3) yields:

$$\frac{\partial a}{\partial s} = \frac{4s+7a}{9(2+\rho)-7s-12a} \in (0,1)\cdot$$

(i) Using this result, it is straightforward to see that $\frac{\partial p_2}{\partial s} < 0$. Taking derivatives,

$$\begin{aligned}\frac{\partial p_2}{\partial s} &= -\frac{1}{3}\left(\left(1-\frac{\partial a}{\partial s}\right)\left(x_1-\frac{1}{2}\right)+\frac{\partial a}{\partial s}\right) \\ &\quad -\frac{1}{3}\frac{1}{1+\rho}\left(\frac{2}{3}\left(\frac{s}{3}+\frac{a}{2}\right)+\left(1-\frac{s}{3}\right)\left(\frac{1}{3}+\frac{1}{2}\frac{\partial a}{\partial s}\right)\right).\end{aligned}$$

(ii) Taking derivatives,

$$\begin{aligned}\frac{\partial p_1}{\partial s} &= \frac{1}{3}\left(\left(1-\frac{\partial a}{\partial s}\right)x_1-\frac{1}{2}\left(1+\frac{\partial a}{\partial s}\right)\right) \\ &\quad -\frac{1}{3}\frac{1}{1+\rho}\left(\frac{2}{3}\left(\frac{s}{3}+\frac{a}{2}\right)+\left(1-\frac{s}{3}\right)\left(\frac{1}{3}+\frac{1}{2}\frac{\partial a}{\partial s}\right)\right).\end{aligned}$$

The second term is negative, while the sign of the first term cannot be determined in general. Solving for $x_1$, expression above is positive if and only if

$$x_1 > \widehat{x}_1(s) = \frac{1}{\left(1-\frac{\partial a}{\partial s}\right)}\left(\frac{1}{1+\rho}\left(\frac{2}{3}\left(\frac{s}{3}+\frac{a}{2}\right)+\left(1-\frac{s}{3}\right)\left(\frac{1}{3}+\frac{1}{2}\frac{\partial a}{\partial s}\right)\right)+\frac{1}{2}\left(1+\frac{\partial a}{\partial s}\right)\right).$$

The fact that $\widehat{x}_1(s) > \frac{1}{2}$ follows since $\frac{\partial p_1}{\partial s}$ is weakly increasing in $x_1$ and $\frac{\partial p_1}{\partial s} < 0$ for $x_1 = \frac{1}{2}$, as the first term becomes $-\frac{\partial a}{\partial s} < 0$.

(iii) As $s$ increases, the average price changes as follows:

$$\frac{\partial p\,(t)}{\partial s} = \frac{\partial\,(p_1-p_2)}{\partial s}x_1 + (p_1-p_2)\frac{\partial x_1}{\partial s} + \frac{\partial p_2}{\partial s}\cdot$$

The first term is positive given that an increase in $s$ enlarges the difference between the prices charged by the two firms. This follows from the fact that the price differential $p_1 - p_2$ is directly proportional to $s - a$ and, as shown above, $\frac{\partial a}{\partial s} < 1$. However, the second and third terms are negative. Hence, the sign of the effect of $s$ on average prices is ambiguous. In particular, an increase in $s$ leads to a reduction in the average price only when firms are sufficiently symmetric, i.e., if $x_1 < \widetilde{x}_1$. Note that $\widetilde{x}_1 > \widehat{x}_1(s)$ as $x_1 < \widehat{x}_1(s)$ is a sufficient condition for the average price to go down in $s$, as both firms' prices are decreasing in $s$ (part $(ii)$).

## Proof of Lemma 2

(i) It follows from the fact that $\dot{p}(t)$ is inversely proportional to $s - a$ and, as shown above, $\frac{\partial a}{\partial s} < 1$. (ii) The transition to the steady state occurs at a rate which is inversely proportional to $\frac{s+2a}{3}$, and as shown above, $\frac{\partial a}{\partial s} > 0$.

## Proof of Proposition 2

Steady-state prices are

$$\lim_{t\to\infty} p_i(t) = p^* = \frac{1}{2} + \frac{a}{2(1+\rho)} - \frac{s(1+\frac{a}{2})}{3(1+\rho)}.$$

Taking derivatives w.r.t. $s$,

$$\frac{\partial p^*}{\partial s} = \frac{a}{2(1+\rho)}\frac{\partial a}{\partial s} - \frac{1+\frac{a}{2}}{3(1+\rho)} - \frac{s}{6(1+\rho)}\frac{\partial a}{\partial s} < 0,$$

where the inequality follows from $\frac{\partial a}{\partial s} \in (0,1)$ and $a < \frac{s}{2}$.