Dynamic Auctions for On-Demand Services

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***Abstract*—In this paper, we consider a market in which a finite number of firms compete in prices for the incoming demand for service. Upon every customer arrival, an independent auctioneer gathers bids from each one of the competing queuing systems and assigns the incoming customer to the system that submitted the lowest bid. We provide a simple characterization of Markov Perfect equilibrium in terms of "indifference prices," i.e., price levels at which players are indifferent between committing available capacity or withholding it. We identify sufficient conditions for socially efficient performance in equilibrium.**

***Index Terms*—Auctions, dynamic games, Markov Perfect equilibrium (MPE), queuing systems.**

I. Introduction

The pervasive nature of information technology (IT) in modern economies has inevitably changed the underlying structure through which economic transactions take place. As electronic commerce emerges as a viable retail channel, many firms have begun experimenting with alternative trading mechanisms such as auctions, guaranteed-purchase contracts, group purchasing, etc. These new trading mechanisms offer unprecedented opportunities in improving the operational efficiency of capacity-constrained industries. Typically, the online trader or auctioneer is a "retail consolidator," i.e., a firm that either buys cheaply, in advance, major blocks of capacity (e.g., airline seats or hotel rooms) or has access to a surplus inventory of deeply discounted capacity to be traded at possibly marked-up prices. Often, retail consolidators or aggregators sell available capacity "on-demand," i.e., an auction (or other trading mechanism) is launched upon request by a potential customer. In more traditional retail channels, potential customers incur "search" costs in obtaining and processing information about available prices. Similarly, sellers face costs associated with advertising, labor, etc., which is typical of traditional retail activities. By relying on "retail consolidators" and their chosen trading mechanisms, firms can adequately price-in installed capacity while being able to focus on their core business competency. Customers may also save on the information-search costs since the more successful online trading sites are exactly those that develop a reputation for enabling better price discovery. To summarize, on-demand trading has the potential to lower transaction costs and to enable better tracking between market prices and available capacity.

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Our interest in auction-based mechanisms for on-demand service is also motivated by recent proposals for the implementation of a new paradigm known as "utility" computing. This label is generally used to refer to various schemes in which the computing resources of possibly different firms or "application-service providers" are pooled in order to be allocated and priced upon customer request. It is believed that the utility-computing paradigm would induce a more efficient exploitation of the existing IT infrastructure. In addition, by outsourcing IT services on-demand, small users avoid the substantial investment costs associated with owning and operating a separate IT division within their organizations.

In this paper, we explore the foundations of a theoretical model that captures many relevant features of an auction-based structure for on-demand trading of homogeneous services. Specifically, we consider a market in which a finite number of firms compete in prices for attending incoming demand for service. Each firm owns a queuing system (server with finite buffer) to provide the service. These queuing systems operate in parallel and, upon every customer arrival, an independent auctioneer gathers bids from each one of the competing queuing systems and assigns the incoming customer to the system that submitted the lowest bid.

Since only prices (and not capacity) can be updated in the short-run, we focus our attention on strategic pricing. In particular, we characterize the opportunity costs associated to committing available capacity at a given point in time. These opportunity costs are strategic in that they depend not only upon a firm's available capacity at a given point in time but also on competitors' available capacity and the way these competitors price in scarcity. Opportunity costs are also dynamic in that they depend on the "state" of the available capacity in the system. Hence, this setting is not one of repeated auctions but one of dynamic auctions since the underlying opportunity costs that the players are facing vary over time.

The structure of the paper is as follows. In Section II, we briefly review the related literature. In Sections III and IV, we introduce the basic setup and the notion of Markov Perfect equilibrium (MPE). In Section V, we give a simple characterization of equilibrium in terms of "indifference prices," i.e., price levels at which players are indifferent between committing available capacity or withholding it. In Section VI, we study conditions under which systemwide performance in equilibrium is efficient.

II. Literature Review

Our analysis is related to two strands of the literature: The literature that analyzes strategic behavior in queuing systems and the auction literature.

The literature on strategic behavior in queuing systems has been recently surveyed in [3]. In the pioneering work of

Naor [13], arriving customers are assumed to be able to choose whether to join or leave, depending on their expected disutility of delay. A congestion externality is, therefore, characterized as the increased delay due to other users' utilization of the service facility. If a customer enters the service system, then other customers may have to wait longer. Naor's work introduced the use of congestion-pricing methods to alleviate the harmful effects of the congestion externality. Notable extensions to Naor's work can be found in [9] and [12].

Our analysis departs from this line of work in that we are interested in modeling competition among servers. On a chapter of their recent survey, [3, Ch. 8] provide a review of published literature on competition among servers. Some papers (see [5] and [7]) model nonprice competition (e.g., competition in service rates), while a few others, such as [8], study how delay costs affect both prices and operating policies. In all of these papers, firms maximize average reward. Consequently, equilibrium analysis is firmly grounded on (long-run) average reward and costs functions.

Our analysis differs substantially from the aforementioned papers in that, in our model, firms do account for the time value of money when evaluating their decisions. Consequently, the short-run performance of their queuing systems defines total discounted profits. Thus, while models that assume firms maximize average reward may be better suited to evaluate the long-run effects of, for example, installed capacity and/or service rates, models with discounted profit capture the need for dynamically adjusted prices to account for varying opportunity costs.

This paper is also related to the auction literature (see [6] for a complete survey of the auction literature). Most auction models start by asserting assumptions on the valuation structure of bidders. In our setting, players' valuations are not independent, as they are endogenously determined as a result of market interaction. Endogenous valuations arise because players' profits are interrelated across periods through costs, i.e., since buffer sizes are limited, serving the current customer may imply an infinite cost of serving the next incoming customer, and through prices. The prices that the rivals will be willing to charge to the future incoming customers will depend on whether their servers are idle or busy and on how they anticipate future customers will be allocated among players. However, we assume that, given a pricing strategy, every player is able to determine the opportunity costs of his/her opponents. In this sense, our model is one with complete information but endogenous valuations.

Finally, this paper is related to another strand in the literature on load balancing and scheduling for computer-based services (see [2] and [15]).

III. Setup

We now present the basic setup for price-based competition among queuing systems.

A.1) We assume that customers arrive according to a Poisson process with rate λ. The service tasks requested by incoming customers are homogeneous and their willingness to pay, which we shall denote by v, is deterministic. There are n parallel queues (server plus buffer) owned by n different players. We shall denote by μ_i and K_i, the mean service rate (assuming service times are independently identically distributed and exponentially distributed) and buffer capacity of player i's queue. The above summarized structure is assumed to be common knowledge.

A.2) Auctions take place every time customers arrive. Given that arrivals constitute a Poisson process, the probability that two or more consumers arrive at the same time is zero. Hence, when an auction takes place, players will be bidding to service one customer only. Bids can take any value in $[0, v]$ and players with full buffer are not allowed to bid in.[1] Given the bids by players $b = (b_1, b_2, \ldots, b_n) \in B = \Pi_i[0, v]$, we shall denote by $b_{[k]}$ the kth lowest bid, where $[k]$ represents the index of the player submitting that bid. The winner of the auction will be the one that submitted the lowest bid (in case of a tie among the players with the lowest bids, the index [1] is randomly assigned to one of them). The price for the service $p^*(b)$ will be set at the second lowest bid. Formally, player's i demand $D_i(b)$ and the service price are defined as follows:

$$D_i(b) = \begin{cases} 1, & \text{if } b_i = b_{[1]} \\ 0, & \text{otherwise} \end{cases}$$

where ties are randomly broken in case several players bid $b_{[1]}$. In addition

$$p^*(b) = \begin{cases} b_{[2]}, & \text{if } [2] \neq \varnothing \\ b_{[1]}, & \text{otherwise.} \end{cases}$$

The assumption that customers are not sensitive to service quality will be revisited in Section VII.

A.3) The "state" of the system is the number of customers at each buffer. The state space is therefore $\mathcal{X} = \prod_{i=1}^{n}\{0, 1, 2, \ldots, K_i\}$. Players have complete information about the state of the system.

A. Stationary Markovian Pricing Strategies

We shall restrict our attention to Markovian pricing strategies, i.e., strategies where bids are a function of the current state of the system. Moreover, we are interested in stationary (i.e., time invariant) strategies.

For each player i, a pure Markovian pricing strategy is denoted by the mapping $\pi_i : \mathcal{X} \mapsto [0, v]$. A Markovian strategy combination, $\pi = (\pi_1, \pi_2, \ldots, \pi_n)$, is a vector of Markovian pricing strategies for each player. We let $\Pi \subseteq (\mathcal{X} \times [0, v])^n$ represent the set of all Markovian (pure) strategy combinations.

Let us denote by $r_i^\pi(x)$ the player i's expected payoff in the auction played when the state of the system is x and players follow strategy combination π, i.e.,

$$r_i^\pi(x) = p^*\left(\pi(x)\right) D_i\left(\pi(x)\right).$$

We limit our interest to the times at which a new customer arrives. Given that a customer just arrived and conditional upon the next customer arrival occurring after t units of time, we shall denote by $\mathbf{Q}^{\pi,t}$ the one-step transition probability matrix. Specifically, given states $x = (x_1, x_2, \ldots, x_n)$ and $y = (y_1, y_2, \ldots, y_n) \in \mathcal{X}$, the probability of transitioning from

[1] This assumption could be rationalized as follows: if large penalties for defaulting on the auction outcome are implemented, players with full buffer will abstain from bidding.

x to y, given that the next arrival will occur at time t and that the prices prescribed by the strategy π are used, is given by

$$\mathbf{Q}^{\pi,t}(x,y) = \prod_{i=1}^{n} Q_i^{\pi,t}(x,y_i)$$

where $Q_i^{\pi,t} \in \Re^{|\mathcal{X}|\times K_i}$ is a matrix in which the (x,y_i)th entry defines the probability of transitioning from state x to any state whose ith component is y_i, i.e.,

$$Q_i^{\pi,t}(x_i,y_i) = \begin{cases} \bar{Q}_i^t(x_i,y_i), & \text{if } D_i\left(\pi(x)\right)=1 \\ \underline{Q}_i^t(x_i,y_i), & \text{otherwise} \end{cases}$$

and $\bar{Q}_i^t$ is the transition-probability matrix for player i's state variable, if at present time, he/she is to attend the current customer conditioned on the next customer arriving after t units of time, i.e.,

$$\bar{Q}_i^t(x_i,y_i) = \begin{cases} 0, & \text{if } y_i > x_i+1 \\ e^{-\mu_i t}\frac{(\mu_i t)^{(x_i-y_i+1)}}{(x_i-y_i+1)!}, & \text{if } 0<y_i\le x_i+1 \\ \sum_{l>x_i} e^{-\mu_i t}\frac{(\mu_i t)^l}{l!}, & \text{if } y_i=0 \end{cases}$$

and $\underline{Q}_i^t$ is the one-step transition probability vector for player i's state variable, if at present time, he/she is not to attend the current customer, and the next customer arrives after t time units, i.e.,

$$\underline{Q}_i^t(x_i,y_i) = \begin{cases} 0, & \text{if } y_i \ge x_i+1 \\ e^{-\mu_i t}\frac{(\mu_i t)^{(x_i-y_i)}}{(x_i-y_i)!}, & \text{if } 0<y_i\le x_i \\ \sum_{l\ge x_i} e^{-\mu_i t}\frac{(\mu_i t)^l}{l!}, & \text{if } y_i=0\,. \end{cases}$$

With this notation in hand, the value function is denoted by the mapping $V^{\pi,t}:\mathcal{X}\mapsto\mathcal{R}^n$ and can be recursively defined as follows:

$$V_i^{\pi,t} = r_i^\pi + e^{-\rho t}\mathbf{Q}^{\pi,t}V_i^\pi$$

where r_i^π is the immediate reward, the term $e^{-\rho t}\mathbf{Q}^{\pi,t}V_i^\pi$ is the expected (discounted) continuation reward, and $V_i^\pi = E[V^{\pi,t}]$.

1) Example 1—Computing the Transition-Probability Matrix: Suppose that there are $n=2$ parallel queues (server plus buffer) owned by the two different players. Each server has exponential service rate μ and buffer capacity of one. Thus, for $i=1,2$

$$\bar{Q}_i^t = \begin{pmatrix} 1-e^{-\mu t} & e^{-\mu t} & 0 \\ \sum\limits_{k>1} e^{-\mu t}\frac{(\mu t)^k}{k!} & e^{-\mu t}(\mu t) & e^{-\mu t} \\ \sum\limits_{k>1} e^{-\mu t}\frac{(\mu t)^k}{k!} & e^{-\mu t}(\mu t) & e^{-\mu t} \end{pmatrix}$$

$$\underline{Q}_i^t = \begin{pmatrix} 1 & 0 & 0 \\ \sum\limits_{k>0} e^{-\mu t}\frac{(\mu t)^k}{k!} & e^{-\mu t} & 0 \\ \sum\limits_{k>1} e^{-\mu t}\frac{(\mu t)^k}{k!} & e^{-\mu t}(\mu t) & e^{-\mu t} \end{pmatrix}.$$

B. Valuation

Let $M_i \subset \mathcal{X}$ be defined as $M_i = \{x\in\mathcal{X}|x_i<K_i\}$. In the following discussion, we restrict our attention to stationary Markovian pricing strategies π_i in the reduced domain M_i, i.e., $\pi_i : M_i \subset \mathcal{X} \mapsto [0,v]$. This is done with no loss of generality, since, at every stage game, players with a full buffer capacity (i.e., $x_i = K_i$) will not participate in the next auction.

If players are assumed to follow a Markovian pricing policy π, from an individual perspective, each player is faced at each state with the choice between selling one unit of capacity at the given price or withholding one unit of capacity. In order to compute the optimal response, we solve the dynamic programming recursive equations conditional upon the next customer arrival occurring in t units of time. For $x\in M_i$ and $b_i\in[0,v]$, we have

$$\begin{aligned}\hat{V}_i^{\pi,t}(x;b) =& \,p^*\left(b_i,\pi_{-i}(x)\right)D_i\left(b_i,\pi_{-i}(x)\right) \\ &+ e^{-\rho t}\sum_{x'\in\mathcal{X}}\mathbf{Q}^{\pi,t}(x,x')\hat{V}_i^\pi(x')\end{aligned} \tag{1}$$

$$\hat{V}_i^\pi(x) = \sup_{0\le b_i\le v} E\left[\hat{V}_i^{\pi,t}(x;b_i)\right] \tag{2}$$

where $(b_i,\pi_{-i}(x))$ stands for the strategy combination that equals π, except at state x, where player i bids b_i.

Equation (1) determines the value of selling today at the given prices (by bidding b_i). Equation (2) summarizes the value of today's best decision.

IV. Markov Perfect Equilibrium

We are interested in the Markovian strategy combinations that have the following property: At every time period, for any given state, no player can do strictly better by choosing a different price than the one prescribed by the strategy combination under consideration. This concept formalizes a notion of recursive rationality, i.e., play prescribed by the strategies from any state off the equilibrium path must also be in equilibrium (see [1]). As a refinement of Nash equilibrium, this solution concept filters out all "noncredible" Nash equilibria, i.e., those equilibrium strategies supported upon the basis of irrational play off the equilibrium path. In light of this, the MPE solution concept has more predictive power than Nash equilibrium. As argued in [10], a second advantage of MPE pertains to the simplicity of Markovian strategies, which substantially reduces the number of parameters to be estimated in dynamic econometric models.

Formally, a strategy combination π^* is an MPE, if and only if, for every player i and every state $x\in\mathcal{X}$

$$V_i^{\pi^*}(x) \ge V_i^{(\pi_i,\pi^*_{-i})}(x)$$

for all $\pi_i \ne \pi_i^*$, where (π_i,π^*_{-i}) is the strategy combination with player i bidding according to π_i (instead of π_i^*).

A. Indifference Prices

To characterize players' best replies for a given strategy combination π, we now introduce the notion of indifference prices. Given state x and conditional upon the next customer arrival occurring in t units of time, we shall denote it by $\tilde{p}_i^{\pi,t}(x)$.

Definition 1: The indifference price map $\tilde{p}_i^{\pi,t}(x) : \Pi \times \mathcal{X} \mapsto [0, v]$ is such that

$$\tilde{p}_i^{\pi,t}(x) + e^{-\rho t} \sum_{x' \in \mathcal{X}} \bar{Q}_i^t(x_i, x_i') \underline{\mathbf{Q}}_{-i}^t(x, x') \hat{V}_i^\pi(x') = e^{-\rho t} \sum_{x' \in \mathcal{X}} \underline{Q}_i^t(x_i, x_i') \mathbf{Q}_{-i}^{\pi,t}(x, x') \hat{V}_i^\pi(x')] \quad (3)$$

where $\underline{\mathbf{Q}}_{-i}^t(x, x') = \prod_{j=1, j\neq i}^n \underline{Q}_j^t(x_j, x_j')$, and $\mathbf{Q}_{-i}^{\pi,t}(x, x') = \prod_{j=1, j\neq i}^n Q_j^{\pi,t}(x_j, x_j')$.

The indifference price for player i is the price that equates the value obtained by selling one unit of buffer capacity today [left-hand side in (3)] and withholding that unit for future revenue [right-hand side in (3)]. We remark that $\tilde{p}_i^{\pi,t}(x)$ is nonnegative. By withholding capacity, a firm ensures that its buffer capacity, next time a customer arrives, will be greater than if it had served the current customer. Since withholding capacity by a given firm causes its rivals to use up their capacity, rivals' buffers will be less (next time a customer arrives) than if that firm were to serve the current customer. Since a firm's value rises as its available capacity increases and rivals' capacities decrease, it must hold that the future value associated with withholding is greater than (or equal to) the future value associated with selling. In addition, $\tilde{p}_i^{\pi,t}(x) \leq v$ since the difference in per unit value between selling and withholding cannot exceed v.

Lastly, the expected indifference prices are denoted

$$\tilde{p}_i^\pi(x) = E\left[\tilde{p}_i^{\pi,t}(x)\right]. \quad (4)$$

1) Example 2—Computing the Indifference Prices: Suppose that there are two parallel queues owned by two different players. Each server has exponential service rate μ and no buffer capacity. Further assume that consumers' willingness to pay is deterministic and equal to v.

In states (1, 0) or (0, 1), when only one of the servers is available, the "available" player has monopoly power. Consequently, dynamic pricing strategies in equilibrium must set prices (for the "available" player) equal to v. The only nontrivial pricing decision is associated with state (0, 0), i.e., both servers are idle. Assuming that equilibrium pricing strategies are symmetric, we denote by $V(x_1, x_2)$ the value function for player 1 when the state of the system is (x_1, x_2) under the (symmetric) equilibrium pricing strategy.

Conditional upon the next customer arrival occurring in t units of time, the value function for player 1 when both servers are busy $V^t(1, 1)$ satisfies the following equation:

$$V^t(1,1) = e^{-\rho t}\left[(1 - e^{-\mu t})^2 V(0,0) + (1 - e^{-\mu t})e^{-\mu t} \times [V(0,1) + V(1,0)] + e^{-2\mu t} V(1,1)\right]. \quad (5)$$

Given that both servers are busy, there are no immediate payoffs but only future expected payoffs that are discounted $e^{-\rho \cdot t}$. The expectation of future payoffs is obtained after considering the four possible transitions: Service times for both busy servers are less than the interval length t, thus the system state transitions to (0, 0); service time for one server is less than t, while the other server's service time exceeds t, hence, the transitions to asymmetric states (0, 1) and (1, 0); and lastly, service times for both busy servers exceed interval length t, thus the system state reverts to (1, 1).

It is easy to see that the value function for player 1 when its server is busy while that of player 2 is available, $V^t(1, 0)$ is equal to $V^t(1, 1)$, as there are no immediate payoffs to be made out of the current customer. Hence

$$V^t(1,0) = V^t(1,1). \quad (6)$$

When player 1's server is available while player 2's is busy, player 1 earns a monopoly rent v on the current customer as it faces no competition from its rival. Hence

$$V^t(0,1) = v + V^t(1,1). \quad (7)$$

Lastly, when both servers are idle, player 1 would be indifferent between providing service at a price $\tilde{p}^t(0, 0)$ or waiting (withholding capacity) if the following condition holds:

$$\tilde{p}^t(0,0) + e^{-\rho t}\left[e^{-\mu t} V(1,0) + (1 - e^{-\mu t}) V(0,0)\right] = e^{-\rho t}\left[e^{-\mu t} V(0,1) + (1 - e^{-\mu t}) V(0,0)\right].$$

Thus

$$\tilde{p}^t(0,0) = e^{-(\rho+\mu)t}\left[V(0,1) - V(1,0)\right].$$

From (6) and (7), we obtain

$$\tilde{p}^t(0,0) = e^{-(\rho+\mu)t} v.$$

The expected value is

$$\tilde{p}(0,0) = E\left[\tilde{p}^t(0,0)\right] = \int_0^\infty e^{-(\rho+\mu)t} v^* \lambda e^{-\lambda t} dt = \frac{\lambda}{\rho + \mu + \lambda} v.$$

V. Characterization, Existence, and Computation of MPE

Theorem 1: A strategy combination of the form $\pi_i(x) = \tilde{p}_i^\pi(x)$, for all i and $x \in \mathcal{X}$, is an MPE.

Proof: We first show that bidding $b_i < \tilde{p}_i^\pi(x)$ is weakly dominated by simply bidding $\tilde{p}_i^\pi(x)$. This deviation has two possible implications.

1) If at state x, given bids (b_i, π_{-i}), player i is the lowest bidder and the second lowest bid is below player i's indifference price, i.e., $p^*(b_i, \pi_{-i}) < \tilde{p}_i^\pi(x)$, then player i's expected payoff is

$$p^*(b_i, \pi_{-i}(x)) + E\left[e^{-\rho t} \sum_{x' \in \mathcal{X}} \bar{Q}_i^t(x_i, x_i') \underline{\mathbf{Q}}_{-i}^t(x, x') \hat{V}_i^\pi(x')\right]. \quad (8)$$

On the other hand, if player i had bid at his expected indifference price, his expected payoff would have simply been

$$E\left[e^{-\rho t} \sum_{x' \in \mathcal{X}} \underline{Q}_i^t(x_i, x_i') \mathbf{Q}_{-i}^{\pi,t}(x, x') \hat{V}_i^\pi(x')\right]. \quad (9)$$

In the light of (3) and (4)

$$p^* (b_i, \pi_{-i}(x)) + E\left[e^{-\rho t} \sum_{x' \in \mathcal{X}} \bar{Q}_i^t (x_i, x_i') \underline{\mathbf{Q}}_{-i}^t (x, x') \hat{V}_i^\pi (x')\right]$$
$$\leq p^* (\pi(x)) + E\left[e^{-\rho t} \sum_{x' \in \mathcal{X}} \bar{Q}_i^t (x_i, x_i') \underline{\mathbf{Q}}_{-i}^t (x, x') \hat{V}_i^\pi (x')\right]$$
$$= E\left[e^{-\rho t} \sum_{x' \in \mathcal{X}} \underline{Q}_i^t (x_i, x_i') \mathbf{Q}_{-i}^{\pi, t} (x, x') \hat{V}_i^\pi (x')\right].$$

Hence, (8) does not exceed (9). In other words, player i would have been better off bidding his indifference price.

2) If by bidding $b_i < \tilde{p}_i^\pi(x)$, player i is the lowest bidder and the second lowest bidder is bidding above player i's indifference price, player i would have been equally well-off bidding at his indifferent price. Similarly, if by bidding $b_i < \tilde{p}_i^\pi(x)$ player i is not the lowest bidder, here again, player i would have been equally well-off bidding at his indifferent price.

Second, we show that bidding $b_i > \tilde{p}_i^\pi(x)$ is weakly dominated by bidding his indifference price. If, at state x, the vector of bids (b_i, π_{-i}) is such that player i is not the lowest bidder, player i's expected payoff is

$$E\left[e^{-\rho t} \sum_{x' \in \mathcal{X}} \underline{Q}_i^t (x_i, x_i') \mathbf{Q}_{-i}^{\pi, t} (x, x') \hat{V}_i^\pi (x')\right] \tag{10}$$

whereas if player i had bid at his expected indifference price, his expected payoff would have been

$$p^*(\pi(x)) + E\left[e^{-\rho t} \sum_{x' \in \mathcal{X}} \bar{Q}_i^t (x_i, x_i') \underline{\mathbf{Q}}_{-i}^t (x, x') \hat{V}_i^\pi (x')\right]. \tag{11}$$

Again, in the light of (4) and the fact that $\tilde{p}_i^\pi(x) \leq p^*(\pi(x))$

$$E\left[e^{-\rho t} \sum_{x' \in \mathcal{X}} \underline{Q}_i^t (x_i, x_i') \mathbf{Q}_{-i}^{\pi, t} (x, x') \hat{V}_i^\pi (x')\right]$$
$$= \tilde{p}_i^\pi(x) + E\left[e^{-\rho t} \sum_{x' \in \mathcal{X}} \bar{Q}_i^t (x_i, x_i') \underline{\mathbf{Q}}_{-i}^t (x, x') \hat{V}_i^\pi (x')\right]$$
$$\leq p^*(\pi(x)) + E\left[e^{-\rho t} \sum_{x' \in \mathcal{X}} \bar{Q}_i^t (x_i, x_i') \underline{\mathbf{Q}}_{-i}^t (x, x') \hat{V}_i^\pi (x')\right].$$

Hence, (10) does not exceed (11). In other words, player i would have been better off bidding his indifference price. ■

In Theorem 1, we have shown that a strategy combination, in which firms bid their expected indifference prices, is an MPE. In other words, a fixed point of the expected indifference price operation is an equilibrium. A question remains on whether such a fixed point exists. In our next result, we show that this is indeed the case.

Theorem 2: Assuming that $n = 2$, there exists a strategy combination of the form $\pi_i(x) = \tilde{p}_i^\pi(x)$ for all i and $x \in \mathcal{X}$.

Proof: We shall prove that the expected indifference price operator is monotone in $\Pi^* \subset [0, v]^{|\mathcal{X}| \times n}$, where

$$\Pi^* = \{\pi \in [0, v]^{|\mathcal{X}| \times n} | \pi_i(x') \geq \pi_i(x)$$

for all i and x', $x \in \mathcal{X}$, such that, $x' \geq x$.

Let us consider the indifference prices associated with a finite number of arrivals, for example, T. If $\pi \leq \pi'$ and $\pi, \pi' \in \Pi^*$, it follows that, for fixed b_i

$$p^*((b_i, \pi_{-i}(x)) D_i (b_i, \pi_{-i}(x))$$
$$\leq p^* \left((b_i, \pi'_{-i}(x)\right) D_i \left(b_i, \pi'_{-i}(x)\right). \tag{12}$$

The value associated with the last arrival is

$$\hat{V}_i^\pi (x; T, T) = \sup_{0 \leq b_i \leq v} \{p^* ((b_i, \pi_{-i}(x)) D_i (b_i, \pi_{-i}(x))\}.$$

In light of (12)

$$\hat{V}_i^\pi (x; T, T) \leq \hat{V}_i^{\pi'} (x; T, T). \tag{13}$$

The value $\hat{V}_i^\pi (x; T-1, T)$, which is associated with the last two arrivals, is given by the suprimum over bids $b_i \in [0, v]$ of

$$p^* ((b_i, \pi_{-i}(x)) D_i (b_i, \pi_{-i}(x))$$
$$+ E\left[e^{-\rho t} \sum_{x' \in \mathcal{X}} \mathbf{Q}^{\pi, t} (x, x') \hat{V}_i^\pi (x'; T, T)\right]. \tag{14}$$

In an analogous fashion to Theorem 1's proof, it can be seen that bidding $E[\tilde{p}_i^{\pi, t}(x; T-1; T)]$, where

$$\tilde{p}_i^{\pi, t}(x; T-1, T) + e^{-\rho t} \sum_{x' \in \mathcal{X}} \bar{Q}_i^t (x_i, x_i') \underline{\mathbf{Q}}_{-i}^t (x, x') \hat{V}_i^\pi (x'; T, T)$$
$$= e^{-\rho t} \sum_{x' \in \mathcal{X}} \underline{Q}_i^t (x_i, x_i') \mathbf{Q}_{-i}^{\pi, t} (x, x') \hat{V}_i^\pi (x'; T, T)$$

is the optimal solution to (14). Given (13), we conclude

$$\sum_{x' \in \mathcal{X}} \left[\underline{Q}_i^t (x_i, x_i') \mathbf{Q}_{-i}^{\pi, t} (x, x') - \bar{Q}_i^t (x_i, x_i') \underline{\mathbf{Q}}_{-i}^t (x, x')\right]$$
$$\times \hat{V}_i^\pi (x'; T, T)$$
$$\leq \sum_{x' \in \mathcal{X}} \left[\underline{Q}_i^t (x_i, x_i') \mathbf{Q}_{-i}^{\pi', t} (x, x') - \bar{Q}_i^t (x_i, x_i') \underline{\mathbf{Q}}_{-i}^t (x, x')\right]$$
$$\times \hat{V}_i^{\pi'} (x'; T, T).$$

Or equivalently

$$\tilde{p}_i^{\pi, t}(x; T-1, T) \leq \tilde{p}_i^{\pi', t}(x; T-1, T).$$

Moreover, $\tilde{p}_i^{\pi, t}(x; T-1, T) \in \Pi^*$ and

$$\hat{V}_i^\pi (x; T-1, T) \leq \hat{V}_i^{\pi'} (x; T-1, T).$$

By finite induction, we can show that

$$\tilde{p}_i^{\pi, t}(x; 0, T) \leq \tilde{p}_i^{\pi', t}(x; 0, T).$$

In the limit

$$\tilde{p}_i^{\pi,t}(x) = \lim_{T\to\infty} \tilde{p}_i^{\pi,t}(x;0,T) \leq \lim_{T\to\infty} \tilde{p}_i^{\pi',t}(x;0,T) = \tilde{p}_i^{\pi',t}(x).$$

Finally, the result follows by invoking Tarski's fixed-point theorem for the monotone map $\tilde{p}^{\pi}$ on the complete lattice Π^*. ■

Theorem 2 motivates a very simple algorithm for the computation of MPE. Let $\pi_i^0(x) = v$ for all i and $x \in \mathcal{X}$. The algorithm's basic iteration is defined as follows:

$$\pi_i^{k+1}(x) = \tilde{p}_i^{\pi^k}(x).$$

By monotonicity, the sequence $\{\pi^k : k \geq 0\}$ is monotone, which is decreasing with zero as lower bound. Therefore, there exists a limit point $\pi^* = \lim_{k\to\infty} \pi^k$. Finally, it follows that such limit point is a fixed point of the indifference price map, i.e.,

$$\pi_i^*(x) = \tilde{p}_i^{\pi^*}(x)$$

for all i and $x \in \mathcal{X}$.

A. First-Price Auction

As a corollary to Theorem 1, we consider now the case when the auction format is slightly altered so that the auction winner is paid according to his/her bid. In practice, the first-price auction rule is more widely used than the second-price format. In this setting, the player with the lowest expected indifference price has an incentive to "undercut" the player with the second lowest expected indifference price. We shall assume, as is standard in the literature, that in case of bidding ties, demand is served by the player with the lower expected indifference price (in case of ties among players with equal indifference prices, we shall assume that both players face an equal probability of serving demand). For a given strategy combination of π and a given state x, the optimal "undercutting" price is given by

$$\underline{p}^{\pi}(x) = \arg\max_{p \leq \tilde{p}^{\pi}_{[2]}(x)} \left[\left(p - \tilde{p}^{\pi}_{[1]}(x)\right)(1 - F(p))\right].$$

Corollary 1: Given $x \in \mathcal{X}$ and π, let $[k]$ denote the index associated with the kth lowest expected indifference price. If π is of the form

$$\begin{aligned} \pi_{[1]}(x) &= \underline{p}^{\pi}(x) \\ \pi_i(x) &= \tilde{p}_i^{\pi}(x), \qquad i \neq [1] \end{aligned}$$

then π is an MPE.

Proof: By construction, the player with lowest indifference price, i.e., $\tilde{p}^{\pi}_{[1]}(x)$, maximizes expected markup over its indifference price by bidding $\underline{p}^{\pi}(x)$. Since $\underline{p}^{\pi}(x) \leq \tilde{p}^{\pi}_{[2]}(x)$, all the other players have no incentive to deviate. ■

VI. Efficiency

We are interested in studying whether the MPE, which is characterized above, induces a systemwide efficient utilization of resources. Since an incoming customer can only be rejected if all the buffers are busy, a systemwide efficient routing policy is one that maximizes the expected number of customers served over an unbounded horizon. Hordijk and Koole [4] have shown that any systemwide efficient policy has the following structure: an arriving customer should be assigned to a faster server when that server has a shorter queue. They refer to this type of policy as the "Shorter Queue Faster Server Policy, SQFSP." As it is pointed out in [4], this characterization is incomplete in general but sufficient whenever the servers have identical service-time distributions. In this case, the optimal policy follows a simple "Shortest Queue" rule, incoming customers should be routed to the shortest queue. An application to Hordijk and Koole's characterization is the following result.

Corollary 2: Assume that expected indifference prices are monotone in buffer state, i.e.,

$$x_i \geq x_j \Longrightarrow \tilde{p}_i^{\pi}(x) \geq \tilde{p}_j^{\pi}(x)$$

where $\pi_i(x) = \tilde{p}_i^{\pi}(x)$ for all i and $x \in \mathcal{X}$. With identical service-time distributions, π induces a systemwide efficient routing policy.

Nonetheless, when servers have different service-time distributions, the performance of the auction-based controlling mechanism may be inefficient, as we shall illustrate in our next example.

1) Example 3—MPE May not Be Efficient: Again, suppose that there are two bufferless queues owned by two different players and that consumers' willingness to pay is deterministic and equal to v. At state (1, 1), requests will be denied, while at states (0, 1) and (1, 0), the requests will be accepted at a price v. Therefore, the only interesting state is (0, 0). As in example 2 above, it is possible to state a set of equations for the total discounted value for each player. We shall denote by $V_i^t(x_1, x_2)$ player i's discounted value in equilibrium when the state of the system is (x_1, x_2) and the next customer arrival occurs in t units of time

$$\begin{aligned} V_1^t(1,0) = e^{-\rho t}\big[&(1-e^{-\mu_1 t})(1-e^{-\mu_2 t})V_1(0,0) \\ &+ e^{-(\mu_1+\mu_2)t}V_1(1,1) \\ &+ e^{-\mu_1 t}(1-e^{-\mu_2 t})V_1(1,0) \\ &+ (1-e^{-\mu_1 t})e^{-\mu_2 t}V_1(0,1)\big] \\ V_1^t(0,1) = v + e^{-\rho t}\big[&(1-e^{-\mu_1 t})(1-e^{-\mu_2 t})V_1(0,0) \\ &+ e^{-(\mu_1+\mu_2)t}V_1(1,1) \\ &+ e^{-\mu_1 t}(1-e^{-\mu_2 t})V_1(1,0) \\ &+ (1-e^{-\mu_1 t})e^{-\mu_2 t}V_1(0,1)\big] \\ V_1^t(1,1) = e^{-\rho t}\big[&(1-e^{-\mu_1 t})(1-e^{-\mu_2 t})V_1(0,0) \\ &+ e^{-(\mu_1+\mu_2)t}V_1(1,1) \\ &+ e^{-\mu_1 t}(1-e^{-\mu_2 t})V_1(1,0) \\ &+ (1-e^{-\mu_1 t})e^{-\mu_2 t}V_1(0,1)\big]. \end{aligned}$$

Finally, we write down the equation for the indifference price at state (0, 0)

$$\tilde{p}_1^t(0,0) + e^{-\rho t}\left[(1-e^{-\mu_1 t})V_1(0,0) + e^{-\mu_1 t}V_1(1,0)\right] = e^{-\rho t}\left[(1-e^{-\mu_2 t})V_1(0,0) + e^{-\mu_2 t}V_1(1,0)\right].$$

Conversely, for player two, we have the following equations:

$$\begin{aligned}V_2^t(1,0) &= v + e^{-\rho t}\left[(1-e^{-\mu_1 t})(1-e^{-\mu_2 t})V_2(0,0)\right.\\ &\quad + e^{-(\mu_1+\mu_2)t}V_2(1,1)\\ &\quad + e^{-\mu_1 t}(1-e^{-\mu_2 t})V_2(1,0)\\ &\quad \left. + (1-e^{-\mu_1 t})e^{-\mu_2 t}V_2(0,1)\right]\\ V_2^t(0,1) &= e^{-\rho t}\left[(1-e^{-\mu_1 t})(1-e^{-\mu_2 t})V_2(0,0)\right.\\ &\quad + e^{-(\mu_1+\mu_2)t}V_2(1,1)\\ &\quad + e^{-\mu_1 t}(1-e^{-\mu_2 t})V_2(1,0)\\ &\quad \left. + (1-e^{-\mu_1 t})e^{-\mu_2 t}V_2(0,1)\right]\\ V_2^t(1,1) &= e^{-\rho t}\left[(1-e^{-\mu_1 t})(1-e^{-\mu_2 t})V_2(0,0)\right.\\ &\quad + e^{-(\mu_1+\mu_2)t}V_2(1,1)\\ &\quad + e^{-\mu_1 t}(1-e^{-\mu_2 t})V_2(1,0)\\ &\quad \left. + (1-e^{-\mu_1 t})e^{-\mu_2 t}V_2(0,1)\right]\end{aligned}$$

together with the indifference-price equation

$$\tilde{p}_2^t(0,0) + e^{-\rho t}\left[(1-e^{-\mu_2 t})V_2(0,0) + e^{-\mu_2 t}V_2(1,0)\right] = e^{-\rho t}\left[(1-e^{-\mu_1 t})V_2(0,0) + e^{-\mu_1 t}V_2(1,0)\right].$$

At state (0, 0), the player with the lowest indifference price wins the auction at a price equal to the highest indifference price. If we posit that $\tilde{p}_1^t(0,0) < \tilde{p}_2^t(0,0)$ [i.e., the winner at state (0, 0) is player 1 and the price for service is $\tilde{p}_2^t(0,0)$], two additional equations are added

$$\begin{aligned}V_1^t(1,0) &= \tilde{p}_2^t(0,0) + e^{-\rho t}\\ &\quad \times \left[(1-e^{-\mu_1 t})V_1(0,0) + e^{-\mu_1 t}V_1(1,0)\right]\\ V_2^t(0,0) &= e^{-\rho t}\left[(1-e^{-\mu_1 t})V_2(0,0) + e^{-\mu_1 t}V_2(1,0)\right].\end{aligned}$$

Solving, numerically, the above equations for $\tilde{p}_1^t(0,0)$ and $\tilde{p}_2^t(0,0)$ with the parameter values $v = 1$, $\rho = 0.9$, $\lambda = 1$, and $\mu_1 = 0.75$, the conjecture $\tilde{p}_1^t(0,0) < \tilde{p}_2^t(0,0)$ is verified whenever $\mu_2 > 0.75 = \mu_1$. In conclusion, the player with the highest service rate will also have the highest indifference price due to the higher prospect of future monopoly rents. This results in inefficiency since, by assigning arrivals in state (0, 0) to the server with the lowest service rate, there is higher probability of service rejections.

VII. Incorporating Delay Sensitivity

Up to this point, our model assumes that customers are not sensitive to expected delays. While this could apply to settings in which the disutility of delay is negligible when compared to the surplus associated with a service completion, it is of interest to extend our setup to incorporate delay-sensitive customers. In this section, we provide a numerical illustration of the effects associated to incorporating time-sensitive customers.

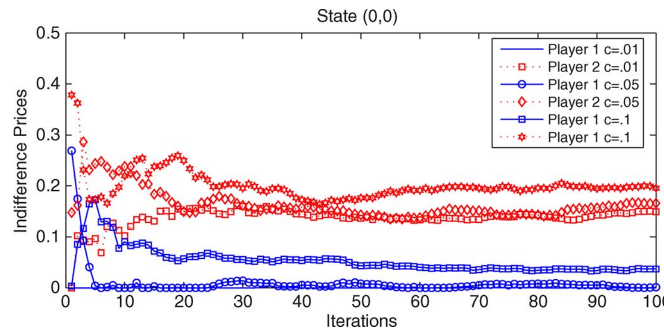

Fig. 1. Evolution of the estimated indifference prices for state (0, 0) with $K_1 = 2$, $K_2 = 4$, and $c \in \{0.01, 0.05, 0.1\}$.

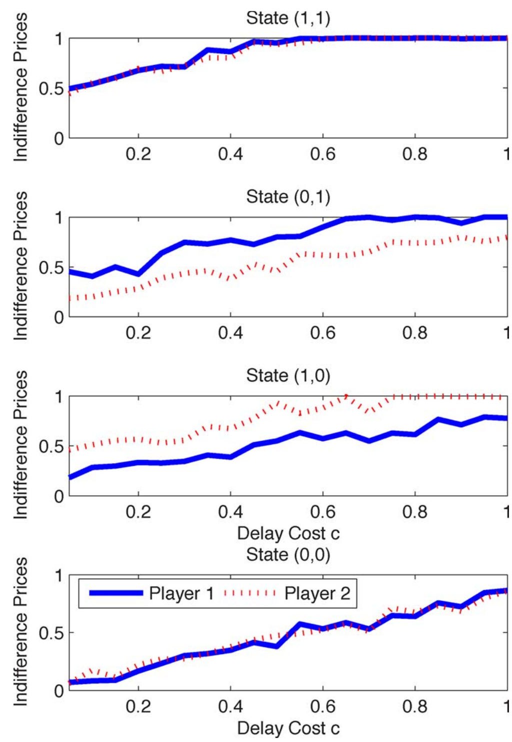

Fig. 2. Plots illustrate the evolution of the indifference prices for all states of the system for the case $K_1 = K_2 = 2$ and for values of the delay cost ranging from $c = [0, 0.05, 0.1, \ldots, 1]$.

A. Setup Revisited

We now revisit assumption A.2) in the basic setup for price-based competition among queuing systems, as presented in Section III.

$A.2'$ Auctions take place every time customers arrive. Arriving customers experience a delay cost c per unit of time. Therefore, given a state $x \in \mathcal{X}$ and bids $b = (b_1, b_2, \ldots, b_n) \in B = \Pi_i[0, v]$, we define [1] to be the index associated with the best offer, i.e.,

$$[1] = \min_i \left\{ b_i + \frac{c x_i}{\mu_i} \right\}$$

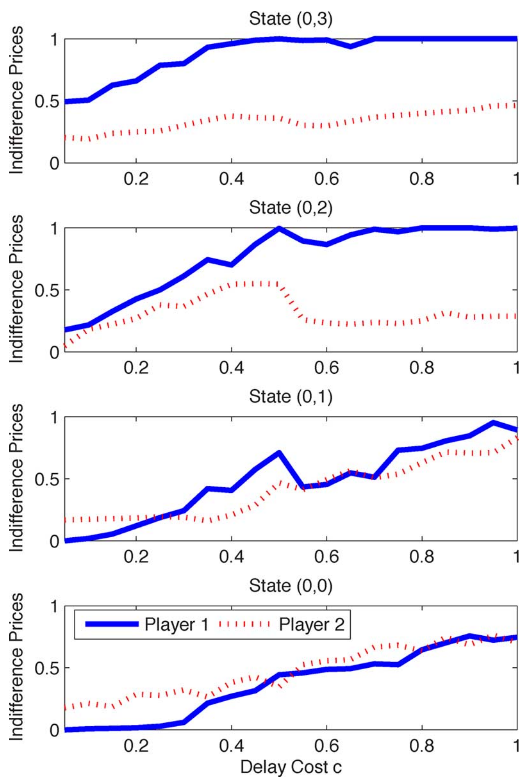

Fig. 3. Plots illustrate the evolution of the indifference prices for all states of the system for the case $K_1 = 2$, $K_2 = 4$ and for delay costs ranging from $c = [0, 0.05, 0.1, \ldots, 1]$.

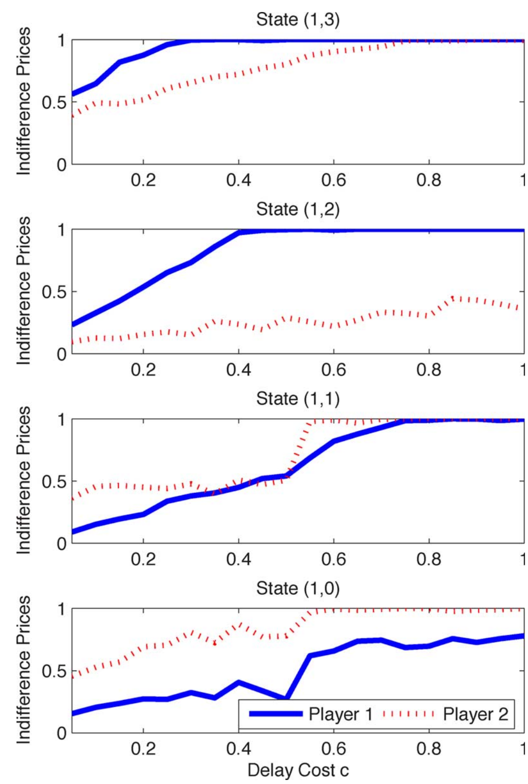

Fig. 4. Plots illustrate the evolution of the indifference prices for all states of the system for the case $K_1 = 2$, $K_2 = 4$ and for delay costs ranging from $c = [0, 0.05, 0.1, \ldots, 1]$.

where ties are randomly broken. Player's i demand $D_i(b, x)$ is defined as follows:

$$D_i(b,x) = \begin{cases} 1, & \text{if } i = [1] \\ 0, & \text{otherwise}. \end{cases}$$

The price for service $p^*(b)$ is defined as follows:

$$p^*(b) = \begin{cases} b_{[2]}, & \text{if } [2] \neq \varnothing \\ b_{[1]}, & \text{otherwise} \end{cases}$$

where

$$[2] = \min_{i \neq [1]} \left\{ b_i + \frac{c x_i}{\mu_i} \right\}.$$

Given the Markovian structure of pricing policies, the definition of indifferent prices given in (3) remains unaltered. The interplay between different pricing strategies and the expected delays is captured by the value-function definition in (1) and (2).

B. Numerical Illustration

Consider a realization of the queuing system, defined by the sequence of states $\{x^1, x^2, \ldots\}$, with $x^k \in \mathcal{X}$ for all $k = 1, 2, \ldots$ occurring at times $\{t_1 < t_2 < \ldots\}$ obtained through simulation of policy $\pi \in [0, v]^{|\mathcal{X}| \times n}$, with initial state x^0. Let $\{k_m\}_{m=1}^{\infty}$ be the subsequence of system states corresponding to a new arrival. Given a roll-out horizon $\tau > 0$, we define the rollout estimate of the value function for state $x = x^0 \in \mathcal{X}$ as

$$\tilde{V}_i^{\pi}(x, \tau) = \sum_{j=1}^{\tau} e^{-\rho(t_{k_j})} r_i^{\pi}(x^{k_j})$$

where $r_i^{\pi}(x^{k_j}) = p^*(b) D_i(b, x^{k_j})$ as defined in the previous section. Using these approximations, player i can estimate its indifference price by letting

$$\hat{p}_i^{\pi}(x) = \tilde{V}_i^{\pi}(x, \tau) - \tilde{V}_i^{\pi}(x + e_i, \tau)$$

where e_i is the unit vector in the ith dimension, defined for all states $x_i < K_i$. We note that this difference is motivated by (3), for the definitions of reward and demand introduced in the previous section.

Let $\pi^0 \in [0, v]^{|\mathcal{X}| \times n}$ be an initial bidding policy, and defined, recursively, as

$$\pi^{k+1}(x) = \max\left\{0, \min\left\{v, (1 - \gamma_k)\pi^k(x) + \gamma_k \hat{p}_i^{\pi^k}(x)\right\}\right\} \tag{15}$$

for $k = 1, 2, \ldots$, and the standard conditions $\sum_k \gamma_k = \infty$, and $\sum_k \gamma_k^2 < \infty$, e.g., $\gamma_k = (1/k)$. Clearly, any fixed point of this recursion corresponds to an MPE. In Fig. 1, we show the results of 100 iterations of the algorithm when applied to a nonsymmetrical system with $K_1 = 2$ and $K_2 = 4$, showing the apparent convergence of the indifference prices.

We applied our algorithm to a symmetrical system with $K_1 = K_2 = 2$, an arrival rate of $\lambda = 1$, service rates of $\mu_1 = \mu_2 = 3/4$, and a roll-out horizon $\tau = 100$. In Fig. 2, we present the summary of different runs for values of the delay cost c ranging from $[0, 0.05, 0.10, \ldots, 0.95, 1.0]$. For each value of c, 100 iterations of the roll-out algorithm were carried out, and the last set of indifferent prices, i.e., $\pi^{100}(x)$, is shown in the figures for each possible state $x \in \{(0,0),(1,0),(0,1),(1,1)\}$, omitting those states where the indifference price is known in advance. The experiment results reflect the symmetry of the setup and also show that, although price increases with the delay cost, the MPE is not necessarily efficient. Figs. 3 and 4 show a similar behavior: 1) the indifference prices increasing with the delay cost and 2) the system is not efficient since the queue with the lowest occupancy will always profit from letting the other player's queue fill up and realize the monopoly price v; this is confirmed by the prices increasing in the other player's queue size (Fig. 3).

VIII. Conclusion

In this paper, we have developed a theoretical model that captures many relevant features of an auction-based structure for online trading of homogeneous services. A simple characterization of MPE in terms of "indifference prices," i.e., price levels at which players are indifferent between committing available capacity or withholding it from the market, reflects the assessment of opportunity costs associated to available capacity at a given point in time. Interestingly, strategic effects are nonnegligible since these opportunity costs not only depend on each firm's available capacity at a given point in time but also on the competitors' available capacity. Therefore, we have a market in which a firm's pricing strategy determines, partially, its competitors' costs. This feature inevitably affects social efficiency. We give a sufficient condition under which systemwide performance in equilibrium is efficient. The incorporation of delay-sensitive demand and relaxing the assumption on complete information about the state of the system are topics of further research.

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